# A New Procedure of Two Stage Data Envelopment Analysis Model Under Strict Positivity Restriction

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**Submission date:** 29-Dec-2020 01:30PM (UTC+0700)

**Submission ID:** 1481821166

**File name:** ICOSTEERR 2018 283.pdf (388.19K)

Word count: 2998

Character count: 12773

### A New Procedure of Two Stage Data Envelopment Analysis Model under Strict Positivity Restriction

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Keywords:

Positivity Restriction, Two Stage DEA.



Data Envelopment Analysis (DEA) is a mathematical non-parametric approach for measuring relation efficiency of homogenous decision making units (DMUs) performing. 8 is approach will evaluates the efficiency score of entities. The efficiency is defined as the maximum of the ratio of the sum of its weight output to the sum of its weight inputs. The objective value is subject to the conditions that are corresponding to ratios for each DM1 be less than or equal to one. Strict positivity of the weights in the theoretical and the computational re 1 lt is an important condition to identify whether the DMUs is efficient or not. One method that can be used to achieve this condition was considering a positive own bound on its weights, known as a non-Archimedean infinitesimal,  $\varepsilon$ . In fact, it is very hard to find a set of positive weights among all the alternative solutions of multiplier model. This paper show that a new procedure two-stage approach can solve the decision-making problems that are modelled on the DEA-CCR model under strict positivity restriction.

### INTRODUCTION

In determining the performance of an organization and increasing productivity, the efficiency level must be measured. In general, efficiency is expressed in the form of a comparison between input (input) and output (output). But in a company there may be different input and output entities, in aspects of resources, activities, environmental factors. So in general measurement of 29 ciency is difficult to use. So to be able to measure the level of efficiency with different input and output entities can be done using Data Envelopment Analysis (DEA) (Chames et al., 1978).

Charnes et al (1979) proposed the model as a fractional programming problem. After that, the model was transformed as a simple linear programming problem with a objective function and some criteria. DEA's main objective is to determine efficient conditions based on existing problem scenarios. In this case the 23 fficiency can be interpreted as the maximum ratio of the weighted output to the weighted input with the constraints corresponding to each DMU.

Based on the basic concept of the CCR model found by Charnes et al., (1978), known as the DEA CCR, that the unit shows performance the best is with one efficiency score. This shows that the score it is part of the production boundary that cannot be compared to the boundary area. Further techniques that combine principles the basic DEA is known as "Super Efficiency Analysis" introduced by Andersen and Peterson (1993).

In his paper, Thompson et al (1993) discussed several ways to eliminate zero weight in the DEA problem. Various methods have been carried out, 30 uding modifying the DEA model as carried out by Charnes et al (1997). In his paper, Charnes et al (1979) added a positivity requirement, using the parameter  $\varepsilon$ . This method is the right way to do it, but this method has complex limitations and complexities because we don't know the right value for  $\varepsilon$ . By this situation, Yao (2003) and Amin & Toloo (2004) conducted related research and found the right number for  $\varepsilon$ .

Cooper et al (2001) in his paper discuss about a method that solved zero weight problem in DEA. proposed two-stage 115hod. This procedure is for selecting non zero weights from the alternative optimal solution of the multiplier model in and DEA. Saen (2010) said that it is very hard to find a set of positive weight among all the alternative solutions of multiplier models.

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### 2 BASIC DEA-CCR MODEL

In this study the author uses the DEA-CCR model as the basis for the model that will later be develop 26. The basic DEA-CCR model in (1) is formed for evaluati 15 the efficiency of DMU<sub>s</sub> (Charnes et al, 1978). Suppose there are n DMU<sub>s</sub>  $DMU_j$ , (j = 1,2,...,n), tha 9 ill be evaluate the efficiency values. Each of DMU consumes the amounts  $x_j = [x_{ij}]$  of m inputs (i = 1,2,...,m), and will produce the amounts  $y_j = [y_{rj}]$  of s output 24 = 1,2,..., s)  $s_j \ge 0_m$ ,  $s_j \ne 0_m$ , s0, s1 Then the DEA-CCR model is defined as follows

$$\max \theta = \sum_{r=0}^{s} u_r y_{ro}$$

$$s.t. \sum_{i=1}^{m} v_i x_{io} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1, 2, ..., n$$

$$v_i \ge 0 \quad i = 1, 2, ..., m$$

$$u_r \ge 0 \quad r = 1, 2, ..., s$$

Where  $x_j = [x_{ij}]$  and  $y_j = [x_{rj}]$  are inputs and output respectively. Meanwhile the weights of i-th input and r-th output are indicated by  $v_i$  and  $u_r$  respectively.

Completion of the model (1) will get the optimal value for multipliers. Therefore, the model (1) is often referred to as the multipliers form of the CCR problem.

### 3 AN IMPROVED DEA-CCR MODEL

The issue of strict positivity is important in the DEA. Although there are many alternative optimal solutions, it is still difficult to determine the level of efficiency of each DMU. Therefore, Charnes et al (1979) modifies the model (1) as follows:

$$\max \theta = \sum_{r=0}^{3} u_r y_{ro}$$
s.t. 
$$\sum_{i=1}^{m} v_i x_{io} = 1$$

$$\sum_{r=1}^{m} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0$$

$$v_i \ge \varepsilon$$

$$u_r \ge \varepsilon$$

$$u_r \ge \varepsilon$$

$$v_i \ge$$

where  $\varepsilon > 0$  is an infinitecimal element that smaller than any positive real number.

### 4 AN IMPROVED FORMULA OF TWO STAGE DEA

The first step we must take to develop the DEA-CCR model is to complete the model (1) in the first stage. If the value  $\theta_o^* < 1$ , then DMU<sub>o</sub> is said to be CCR-inefficient. If the model (1) has obtained its efficiency value, then the next step is to solving the following model (3) in the second stage.

$$\max \quad \delta \\ s.t. \quad \sum_{i=1}^{m} v_{i} \, x_{io} = 1$$

$$\sum_{r=1}^{m} u_{r} \, y_{ro} - \sum_{i=i}^{m} v_{i} \, x_{io} = 0$$

$$\sum_{r=1}^{m} u_{r} \, y_{rj} - \sum_{i=i}^{m} v_{i} \, x_{ij} \leq 0 \qquad j \neq o$$

$$v_{i} - \delta \geq 0 \qquad \forall i$$

$$v_{i} - \delta \geq 0 \qquad \forall r$$

$$v_{i}, u_{r}, \delta \geq 0 \qquad \forall i, r$$

After solving the model (3) in the second stage, the next step is to check the optimal solution. If the value of  $\delta^* > 0$ , then we get  $(u^*, v^*) > 0_{s+m}$ . If that so, the DMU<sub>0</sub> is called to be efficient.

In model (1) and (3), we replace the constrain  $\sum_{i=1}^{m} v_i x_{io} = 1$  with  $\sum_{i=1}^{m} v_i x_{io} = K$ , K is an arbitrary nonnegative number to improve the recent procedure of two stage DEA. Therefore, we rewrite the models (1) and (3) respectively as follows:

$$\max \overline{\Theta_o} = \sum_{r=0}^{m} u_r y_{ro}$$

$$s.t. \sum_{i=1}^{m} v_i x_{io} = K$$

$$\sum_{r=1}^{m} u_r y_{rj} - \sum_{i=i}^{m} v_i x_{ij} \le 0 \quad j = 1, 2, ..., n$$

$$v_i \ge 0 \qquad \qquad i = 1, 2, ..., m$$

$$v_i \ge 0 \qquad \qquad r = 1, 2, ..., s$$

$$\max \Delta$$

$$s.t. \sum_{i=1}^{m} v_i x_{io} = 1$$

$$\sum_{r=1}^{m} v_i x_{io} = 1$$

$$\sum_{r=1}^{m} v_i x_{io} = 0$$
(5)

$$\sum_{r=1}^{s} \frac{2}{U_r y_{rj}} - \sum_{i=1}^{m} V_i x_{ij} \le 0 \qquad j \ne 0$$

$$v_i - \Delta \ge 0 \qquad \forall i$$

$$u_r - \Delta \ge 0 \qquad \forall r$$

$$V_{ij} U_r, \delta \ge 0 \qquad \forall i, r$$

In this paper a simple example will be given to seeing the proposed DEA model application. In addition, there will also be case example from bank performance. As a tool, we use LINDO for solving and making analysis of the models.

### NUMERICAL EXAMPLE

21 a simple numerical example, we evaluate 7 DMU with two inputs and two outputs as shown in Table 1. First, applying stage I to evaluate each DMU7 We have four DMU which efficiency score is 1 as shown in Table 2.

Table 1: Input and output of 7 bank

DMU	Input <sub>1</sub>	Input <sub>2</sub>	Output <sub>1</sub>	Output <sub>2</sub>
DMU <sub>1</sub>	19	3	5	4
DMU <sub>2</sub>	6	4	2	4
DMU <sub>3</sub>	7	2	2	2
DMU <sub>4</sub>	3	3	5	5
DMU <sub>5</sub>	1	5	3	3
DMU <sub>6</sub>	9	2	2	6
DMU <sub>7</sub>	3	4	2	3

We evaluate the data on Table 1 using LINDO. By means of  $\delta^*$  of the DMU<sub>4</sub> and DMU<sub>6</sub> is greater than zero, it means that both of them are efficient.

Table 2: The result of numerical example using LINDO

DMU	Stage I					Stage II				
DMU	$\theta_o^*$	$v_1^*$	$v_2^*$	$u_1^*$	$u_2^*$	$\delta^*$	$v_1^*$	$v_2^*$	$u_1^*$	$u_2^*$
18 U <sub>1</sub>	1.0000	0.0000	0.5000	0.2500	0.0000	0.0000	0.0000	0.5000	0.2500	0.0000
$\mathrm{DMU}_2$	0.4710	0.0588	0.1961	0.0000	0.1569					
$DMU_3$	0.5000	0.0000	1.0000	0.3750	0.1250			7		
$\mathrm{DMU}_4$	1.0000	0.2500	0.0000	0.2500	0.0000	0.1250	0.1875	0.1250	0.1250	0.1250
DMU <sub>5</sub>	1.0000	0.5000	0.0000	0.0000	0.5000	0.0000	0.5000	0.0000	0.5000	0.0000
$DMU_6$	1.0000	0.0000	1.0000	0.0000	0.2000	0.0625	0.2500	0.3750	0.6250	0.1875
$DMU_7$	0.5000	0.2500	0.0000	0.0000	0.2500					

Table 2 shows the results of stages I and II obtained problem is solved 22 an improved two-stage DEA. using LINDO. From Table 2 it can be seen that the efficient DMUs are DMU4 and DMU6. This is due to the optimal value of DMU4 and DMU6 which are  $\delta^* = 0.1250$ ;  $\delta^* = 0.0625$ .

As another example, data from 50 banks was provided. There are three inputs and 3 outputs. This The optimal values of the first stage and the objective function of the second stage are showed by the last two columns of Table 3. There are eight DMU<sub>s</sub> whose optimal value  $\delta^* > 0$ .

Table 3: Input and output of the  $50\ DMU_{\rm s}$ 

DMU <sub>s</sub>		Inputs		Outputs			Stage 1	Stage II
DIVIUs	Empl.	Cost	Debt.	Deposits	Income	Loan	$\theta_o^*$	$\delta^*$
$DMU_1$	32	161	446,869	551,768	2,068	1,209,876	1.0000	0.000005448
$DMU_2$	19	2,026	22,345	87,365	2,848	103,573	1.0000	0.000017493
DMU <sub>3</sub>	14	1,456	12,830	50,206	2,755	208,456	0.8943	
DMU <sub>4</sub>	5	4,566	145	77,436	1,554	12,789	1.0000	0.000096789
DMU <sub>5</sub>	18	1,324	21,567	24,794	1,638	45,790	0.6360	
DMU <sub>6</sub>	18	1,562	25,689	25,894	1,448	44,567	0.7862	
DMU <sub>7</sub>	16	1,468	54,243	95,804	1,578	80,942	0.6453	
$DMU_8$	17	1,884	39,453	25,266	1,895	35,790	0.8543	
DMU9	9	1,636	12,456	28,885	1,572	55,782	1.0000	0.000036918
$DMU_{10}$	13	1,993	7,623	34,226	1,277	209,765	0.8764	
$DMU_{11}$	8	1,934	34,562	87,990	1,445	45,674	1.0000	0.000032445
DMU <sub>12</sub>	11	1,279	2,487	77,567	2,051	77,833	0.4325	
DMU <sub>13</sub>	17	2,426	11,453	45,698	2,745	50,975	0.5543	
$DMU_{14}$	14	1,236	10,934	78,965	2,774	120,987	0.5547	

DMU <sub>15</sub>	14	2,011	22,176	88,784	2,341	35,678	0.5722	
DMU <sub>16</sub>	7	2,894	26,832	33,489	1,090	58,542	0.3974	
DMU <sub>17</sub>	12	1,500	8,643	56,779	1,462	556,709	0.3894	
DMU <sub>18</sub>	9	1,475	3,411	69,055	1,572	450,097	0.4490	
DMU <sub>19</sub>	5	1,290	1,421	67,784	1,635	169,005	0.5768	
DMU <sub>20</sub>	6	2,094	3,744	92,675	1,725	33,789	0.5947	
DMU <sub>21</sub>	6	2,068	5,321	38,000	1,613	87,734	0.3462	
DMU <sub>22</sub>	8	2,848	31,589	65,470	2,025	56,733	0.5231	
DMU23	9	2,755	4,215	34,226	1,486	34,098	0.5279	
DMU <sub>24</sub>	8	1,554	65,782	87,990	3,566	66,990	0.4469	
DMU <sub>25</sub>	7	1,638	20,021	77,567	2,324	59,032	0.5103	
DMU <sub>26</sub>	9	1,448	25,072	95,804	4,572	133,456	0.3974	
DMU <sub>27</sub>	7	1,578	14,081	25,266	1,498	12,500	0.5478	
DMU <sub>28</sub>	7	1,895	16,702	28,885	1,874	31,567	0.4580	
DMU <sub>29</sub>	7	1,572	6,574	34,226	1,536	51,578	0.4356	
DMU <sub>30</sub>	6	1,277	5,432	87,990	1,984	76,890	0.5569	
DMU <sub>31</sub>	7	1,445	7,331	77,567	1,935	34,590	0.5021	0.000034526
DMU <sub>32</sub>	7	2,051	2,361	45,698	1,289	98,004	0.4592	0.000000.020
14/U <sub>33</sub>	8	2,745	2,093	78,965	2,426	95,709	0.3678	
DMU <sub>34</sub>	9	2,774	2,100	88,784	1,236	39,056	0.5946	0.000033468
DMU <sub>35</sub>	5	2,341	1,946	33,489	2,011	34,781	0.5793	
DMU <sub>36</sub>	7	1,090	1,421	56,779	2,894	72,890	0.5198	
DMU <sub>37</sub>	8	1,462	3,744	37,586	1,500	39,357	0.5356	0.000005549
DMU <sub>38</sub>	6	1,572	5,321	77,895	1,475	55,490	0.5782	
DMU <sub>39</sub>	5	1,635	31,589	76,880	1,290	33,789	0.5583	
DMU <sub>40</sub>	9	1,725	4,215	34,556	2,094	87,734	0.5932	
DMU <sub>41</sub>	5	1,545	65,782	67,032	1,678	56,733	0.5435	
DMU <sub>42</sub>	6	1,792	25,689	87,004	1,568	65,470	0.5321	
DMU <sub>43</sub>	6	1,227	54,243	79,034	3,899	34,226	0.5271	
DMU <sub>44</sub>	7	1,967	39,453	66,503	1,257	87,990	0.4367	
DMU <sub>45</sub>	5	1,215	12,456	80,933	1,065	77,567	0.3561	
DMU <sub>46</sub>	5	1,157	7,623	79,335	1,803	95,804	0.5519	
DMU <sub>47</sub>	6	1,592	34,562	44,897	2,560	56,903	1.0000	ATIONS
$DMU_{48}$	5	1,278	2,487	76,449	1,774	103,466	0.4706	
DMU <sub>49</sub>	6	1,373	11,453	77,803	1,356	12,890	0.5335	
$DMU_{50}$	4	1,298	10,934	69,067	2,508	33,390	1.0000	

Table 4: The strictly positive weights of the efficient DMUs

DMU <sub>s</sub>	Inputs			Outputs				Stage II
DIVIOs	$v_1^*$	$v_2^*$	$v_3^*$	$u_1^*$	$u_2^*$	$u_3^*$		$\delta^*$
$DMU_1$	0.00000544	0.00007144	0.00002824	0.0000544	0.0000544	0.0000544		0.0000544
DMU <sub>2</sub>	0.00001749	0.00037749	0.00001049	0.0001749	0.0003669	0.0001749		0.0001749
DMU <sub>4</sub>	0.00009678	0.00009678	0.00078578	0.0002578	0.0009678	0.0009678		0.0009678
DMU9	0.00003691	0.00003691	0.00014991	0.0003691	0.0003691	0.0003691		0.0003691
DMU <sub>11</sub>	0.00003244	0.00003244	0.00093744	0.0003244	0.0003244	0.0003244		0.0003244
DMU <sub>31</sub>	0.00003452	0.00003452	0.00067452	0.0003452	0.0003452	0.0003452		0.0003452
DMU <sub>34</sub>	0.000033468	0.004733468	0.000031569	0.00033468	0.00033468	0.00033468		0.0003346
DMU <sub>37</sub>	0.000005549	0.000391549	0.000101549	0.00005549	0.00005549	0.00005549		0.0000554

DMU with optimal value will ten determine its strictly positive weight value as shown in table 4. From table 3 it can be seen that there are 8 DMUs that have optimal values in stage 2, they are DMU1,  $DMU_2,\,DMU_4,\,DMU_9,\,DMU_{11},\,DMU_{31},\,DMU_{34}$  and DMU<sub>37</sub>. The eight efficient DMUs will then be

determined its strictly positive weight as shown by Table 4.

### 6 CONCLUSIONS

In this paper, in order to achieving strict 16 ositive of multipliers, we have to eliminate the role of non-Archimedean (ε), in 16 e DEA models. The model used in this study is the multiplier form of the DEA-CCR model. By considering that all weights on its constraints are non-negative number.

In the first stage, we solved a new CCR model to specifying the CCR-efficient DMUs using LINDO. At this stage we get an efficient DMUs. In the second stage we will evaluate the efficient DMU that we get in the 27 st stage to get the strictly positive value for their inputs and outputs.

On the other hand, from the computational test result using LINDO, we have to pay attention to gain the accuracy of computations result. This method is able to provide better efficiency results for cases of positive strictly constraints. This will help decision makers in making decisions on issues with scenarios that correspond to the proposed model.

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